Negative Exponents

What do negative exponents mean?

We already know that positive exponents are a way of expressing repeated multiplication. For example:

$$4^3 = 4 \times 4 \times 4 = 64$$

There are a few different ways of thinking about negative exponents, but in general, **negative exponents are the opposite of positive ones.**

All negative exponents can be expressed as their positive **reciprocal**. A reciprocal is a fraction where the numerator and denominator switch places.

$$5^{-3} = \frac{1}{5^3}$$

How can something be flipped into a reciprocal if it wasn't a fraction to start?

We know that numbers can be expressed in more than one way. For example, eight can also be written as:

$$8 = \frac{8}{1}$$

So, negative exponents can be expressed as the positive reciprocal of the base multiplied by itself x times.

$$2^{-3} = \frac{1}{2^3} = \frac{1}{(2)(2)(2)}$$

The larger the negative exponent, the smaller the number it represents. While positive exponents indicate repeated multiplication, negative exponents represent repeated division. That's why 2^-3 is greater than 2^-6.

Flip the base and exponent into the reciprocal, then solve the denominator. Divide the numerator by the denominator to find the final decimal.

Multiplying negative exponents

Good news! The rules for multiplying exponents are the same, even when the exponent is negative.

If the bases are the same, add the exponents. Remember to keep in mind the rules for adding and subtracting negative numbers.

$$4^5 \times 4^{-3} = 4^2$$

If the bases are different but the exponents are the same, multiply the bases and leave the exponents the way they are.

$$7^{-5} \times 6^{-5} = 42^{-5}$$

If there's nothing in common, go directly to solving the equation. Flip the exponents into their reciprocals, then multiply.

$$\begin{aligned} 3^{-2} \times 2^{-3} &= \frac{1}{3^2} \times \frac{1}{2^3} \\ &= \frac{1}{9} \times \frac{1}{8} \\ &= \frac{1}{72} \end{aligned}$$

Dividing negative exponents

Dividing negative exponents is almost the same as multiplying them, except you're doing the opposite: subtracting where you would have added and dividing where you would have multiplied.

If the bases are the same, subtract the exponents.

Remember to flip the exponent and make it positive, if needed.

$$3^{-4} \div 3^{-2} = 3^{-2}$$

$$3^{-2} = \frac{1}{3^2}$$

If the exponents are the same but the bases are different, divide the bases first.

$$8^{-4} \div 2^{-4} = 4^{-4}$$
$$4^{-4} = \frac{1}{4^4}$$

If there's nothing in common, go directly to solving the equation.

$$6^{-2} \div 3^{-3} = \frac{1}{6^2} \div \frac{1}{2^3}$$
$$= \frac{1}{36} \div \frac{1}{27}$$
$$= \frac{1}{36} \times \frac{27}{1}$$
$$= \frac{27}{36}$$
$$= \frac{3}{4}$$
$$= 0.75$$

What happens if the base is negative instead of the exponents?
If the exponent is positive, work with it as you would a regular exponent, but remember two things:

- If the base is negative and the exponent is an even number, the final product will always be **a positive number.**
- If the base is negative and the exponent is an odd number, the final product will always be **a negative number.**

If there are parentheses around the negative base, the power applies to the entire equation -- including the negative sign. If there are no parentheses, the power applies only to the base and not to the negative sign.

Since the first example is being raised to an even power, the two negative signs cancel out and you're left with a positive product. If the exponent was an odd power, the product would be negative because there would be one number that couldn't cancel out.
In the second example, the positive power only applies to the four, not to the negative sign. In this case, the negative sign tells you the product will be negative whether the power is odd or even.

Simplifying negative exponents

Multiplying, dividing and understanding negative exponents is the first step to simplifying expressions with negative exponents.

Remember: all the steps covered above hold true no matter how complicated the expression is.

Let's start by multiplying negative exponents with variables.

$$4x^{-3} = \frac{4}{x^3}$$

In this example, the power only applies to the x base, not the 4. To make it a positive expression, flip the x to the reciprocal and keep the 4 on top.

What about dividing negative exponents with variables?

Let's start with a simple example:

$$\frac{6x^3}{x^{-2}} = 6x^3x^2 = 6x^5$$

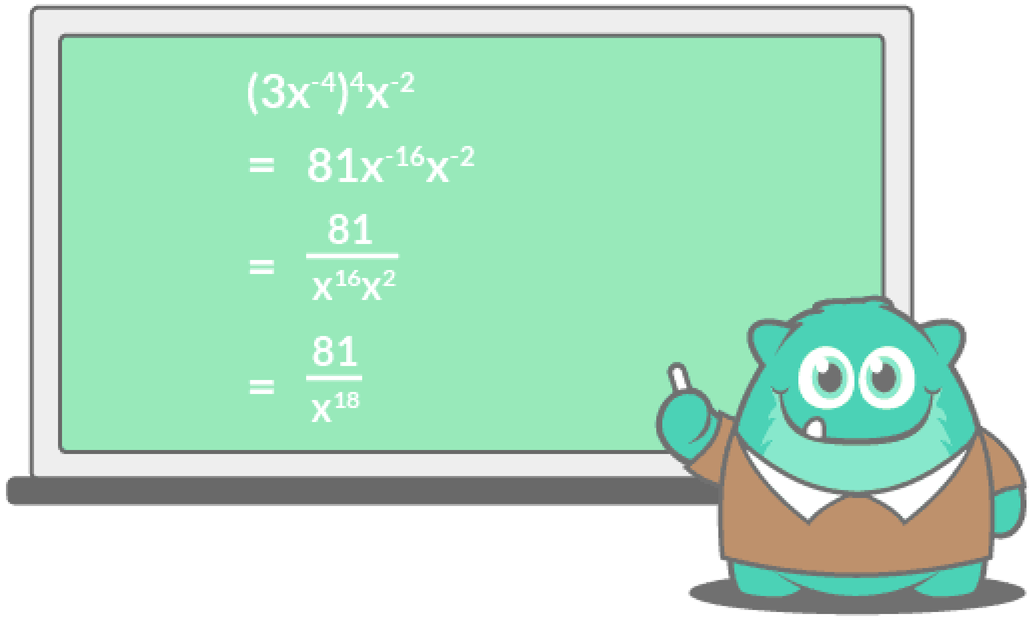
To make the negative exponent positive, move the ???? to the top of the equation and multiply.

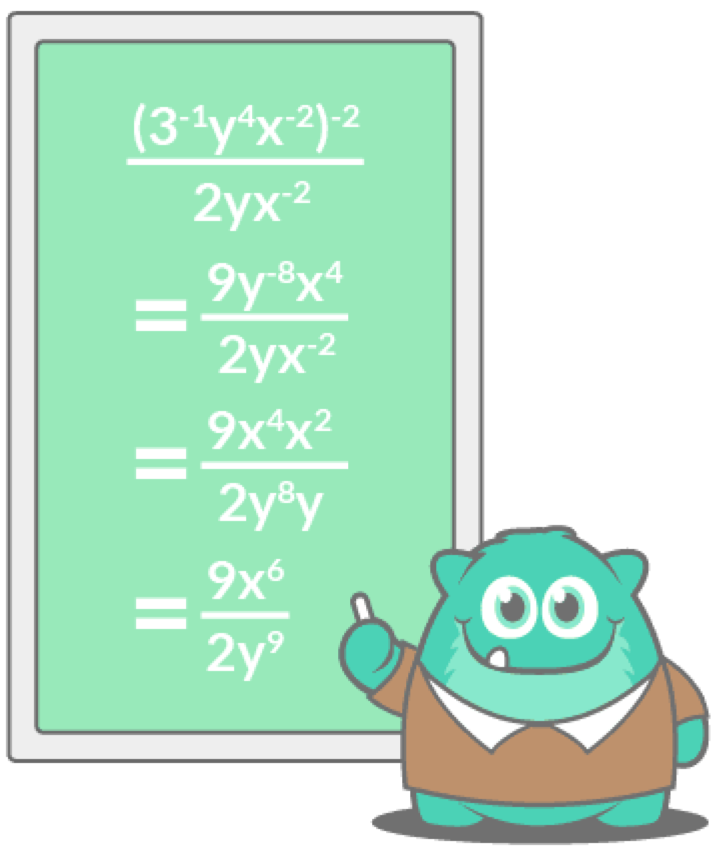
Here's an example of negative exponents with multiple variables:

$$\frac{6x^{-4}}{y^2}$$

Since the negative exponent only applies to the variable, move *x*-4 to the bottom of the equation to make it positive, and leave the 6 where it is.

$$\frac{6}{x^4y^2}$$


$(3x^{-4})^4x^{-2}$
$= 81x^{-16}x^{-2}$
$= \frac{81}{x^{16}x^2}$
$= \frac{81}{x^{18}}$


$\frac{(3^{-1}y^4x^{-2})^{-2}}{2yx^{-2}}$
$= \frac{9y^{-8}x^4}{2yx^{-2}}$
$= \frac{9x^4x^2}{2y^8y}$
$= \frac{9x^6}{2y^9}$

Fractions with negative exponents

We know what to do with whole numbers that have negative exponents, but what about fractions with negative exponents?

To simplify fractions with negative exponents, **flip them to their reciprocals, multiply and reduce**.